THE INHALATION-SHARPENING EFFECT

MARTIN PERGLER

Clarinetist Tony Pay has remarked [1, p. 121] that the pitch of a note played on the clarinet will be slightly sharper if it is played directly after "taking a breath", compared to playing it at the end of a breath-phrase. The subjective manner of playing the note (embouchure, airstream, etc.) remains the same. The difference is slight (much less than a semitone), but noticeable enough that he advocates *not* breathing immediately before an exposed note whose usual fingering pattern "tends sharp" anyway.

Pay qualitatively explains this effect by noting that air exhaled from the lungs is richer in denser than fresh air since it is richer in CO_2. This greater density lowers the speed of sound and thus frequency, compared to "fresh" (atmospheric) air. This has recently been an object of considerable discussion on the Internet [2, Nov.-Dec. 98]. An extreme manifestation occurs if a player burps during playing after drinking carbonated bevarages.

In this article, we explore the physics of this effect (without burping) and theoretically estimate its magnitude (somewhat less than 1/5 of a semitone). It turns out that the change in compressibility of air, as well as its density, are relevant. The discussion involves ideas on an undergraduate level in calculus, physics, chemistry, and human physiology, and the results are applicable to all wind instruments.

1. Breathing and clarinet playing

During inhalation, air enters into the oral cavity through the nose (or mouth). A portion, called *pulmonary* air, is passed into the lungs, where oxygen is partially replaced by carbon dioxide (CO_2). The remainder, called *tidal* air, remains "above" the lungs, where this exchange does not take place. During normal exhalation, the tidal air and (a portion of) the pulmonary air are quickly expelled and replenished during the following inhalation.

When playing a wind instrument, the length of the inhalation-exhalation cycle is prolonged. The player inhales rapidly ("taking a breath") and then exhales slowly and steadily through the instrument over the length of a musical phrase. Pitches (notes) sound when compressional waves at harmonically-related frequencies are induced in the column of exhaled air flowing through the instrument. The pitch or frequency of the sound is determined by the length and shape of this air column (fingering the note) and by the velocity of sound in the air. It is reasonable to suppose that immediately after taking a breath, the air being slowly exhaled will be largely tidal, while later in the phrase (in the same "breath") it will be largely pulmonary. Thus the issue reduces to estimating the difference in velocity of sound in tidal versus pulmonary air. I do not know of direct experimental measurements of this.

2. Method of attack

Atmospheric dry air is a mixture of about 78% N_2, 21% O_2, and 1% other gases, including only trace amounts of CO_2. Exhaled dry air (during normal breathing) has O_2 reduced to about 16% and CO_2 increased to nearly 5%. These percentages are molar fractions (numbers of molecules), which (by Dalton's Law of Partial Pressures) is the same as "composition by volume". Inhaled air

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is warmed to body temperature and humidified largely immediately on its passage through the nose (or mouth), and so the only relevant difference between tidal and pulmonary air will be in the above O_2-CO_2 balance. Thus we will attack the problem by estimating the change in velocity of sound when a molar fraction p of CO_2 is introduced into atmospheric air by replacing a portion of the O_2. The fraction p will be approximately 0.05, pending the discussion at the end.

We make considerable use of the following principle, which is a simple consequence of approximation by differentials in elementary calculus.

Combination of Multipliers. *Suppose a physical quantity is given by an equation* $X = Y^a Z^b$, *where a and b are any fixed real exponents. If Y and Z are multiplied by factors* $(1+\alpha)$ *and* $(1+\beta)$, *where* $|\alpha|, |\beta| << 1$, *then the effect is approximately to multiply X by the factor* $(1 + a\alpha + b\beta)$.

3. Speed of sound in gases

By general principle, the speed of mechanical waves in any medium is given by $v =$[(elastic property)/(inertial property)]$^{1/2}$. In the case of compressional waves in a compressible medium, such as sound, this takes the form $v = [B/D]^{1/2}$. Here D is the density of the gas (or other material). The *bulk modulus B* is the stress/strain ratio of the gas under compression, given in terms of volume and pressure by the equation $B = -\Delta P/(\Delta V/V)$, or $B = -V(dP/dV)$ infinitessimally. The reciprocal $1/B$ is the more intuitive *compressibility* of the gas under pressure.

Since gases are poor conductors of heat, the local thermodynamic changes associated to the propagation of a sound wave are adiabatic (i.e., without heat exchange with the surroundings, or at constant entropy) rather than isothermal. Thus the bulk modulus we require is the one obtained by taking the thermodynamic derivative $(dP/dV)_S$ in the above equation. Calculation gives $B = \gamma P$ and thus $v = (\gamma P/D)^{1/2}$. Here γ is the adiabatic constant of the gas (so that an adiabatic process is characterized by the relationship $PV^\gamma =$ constant).

In our situation, our task is to estimate the change in D and γ between tidal and exhaled pulmonary air; P remains constant at roughly one atmosphere.

4. Change in density

The density of a gas mixture is directly proportional to its average molar weight. The average molar weight of air consisting of 0.79 N_2 (molar weight 14), p CO_2 (molar weight 22), and the balance O_2 (molar weight 16) is $14.4 + 6p$. Denoting by D_p the density of the mixture with CO_2 proportion p, we get

$$D_p = D_0(1 + 0.42p).$$

5. Change in adiabatic constant/compressibility

The adiabatic constant γ is given by the ratio C_P/C_V of specific heats under constant pressure and constant volume, respectively. The molecular-kinetic theory of gases predicts that $C_P = C_V + R$, where R is the universal gas constant, for all ideal pure gases. Furthermore, equipartition for energy predicts that $C_V = (5/2)R$ for diatomic gases. This is borne out by experimental values of about 1.40 for γ of N_2, O_2, and atmospheric air (largely a mixture of diatomic gases).

For polyatomic gases like CO_2, the molecular-kinetic theory and equipartition are no longer sufficient, and it is safer to use experimental values. For CO_2 at room temperature, $C_V = 3.40R$ and $C_P = 4.42R$.

Now consider our 0.79 N_2/p CO_2/rest O_2 mixture. We get that $(C_V)_p = (5/2 + (3.40 - 5/2)p)R = (1 + 0.36p)(C_V)_0$ and $(C_P)_p = (1 + 0.26p)(C_P)_0$ by a similar calculation. From this we see, via combination of multipliers, that

$$\gamma_p = \gamma_0(1 - 0.10p).$$

6. Change in sound velocity and pitch

Since v is proportional to $\gamma^{1/2}D^{-1/2}$, we use our above expressions for γ_p and D_p, together with combination of multipliers, to conclude

$$v_p = v_0(1-0.10p/2-0.42p/2) = v_0(1-0.26p).$$

Frequency of sound is proportional to velocity, and so has the same multiplier. Finally, we convert to a logarithmic pitch scale with 1200 cents per octave (frequency doubling). This is given by $\text{Pitch} = (1200/\log 2)\log f$. We use first-order Taylor approximation to replace $\ln(1+x)$ by x and obtain

$$\text{Pitch}_p = \text{Pitch}_0 - (0.26)p1200/\ln 2 = \text{Pitch}_0 - 450p \text{ cents}.$$

It is clear that changes in density as well compressibility of air (proportional to $1/\gamma$) both contribute appreciably to the change in velocity (and hence pitch), though density dominates by a factor of about 4:1.

7. Estimating the pitch effect

Let us first assume that air vibrating in a clarinet immediately after breathing consists solely of tidal air, that air blown though and vibrating later after a breath consists solely of pulmonary air, and neglect humidity. Then the flattening late in the breath (or sharpening at the beginning) would be exactly as just calculated, plugging in 0.05 for p to get a difference in pitch of 22 cents (about 1/5 semitone). This is comparable to differences in pitch (caused by any number of reasons) routinely consciously noticed and corrected by a capable musician.

There are several reasons why this is likely to be an overestimate. First, all exhaled air is humid, and the calculation was made with data for dry air. Fortunately, both tidal and pulmonary air will be approximately equally humid, so we do not have to account for humidity (or temparature) directly in the calculation of v. However, the presence of water vapour will decrease the relative fractions of both CO_2 and O_2 and hence decrease the value of p (also slightly changing the calculation of γ). However, even at body temparature, the vapour pressure of water is only about 0.06 atmospheres, so this effect will be minimal.

Much more significantly, the vibrating air column inside the clarinet is a mixture into which exhaled air (whether tidal or pulmonary) flows at one end. Thus the value of p inside will be moderated and the overall magnitude of the effect decreased. Furthermore, it should be less pronounced in a flute or recorder, where blown air is split across a sharp edge and only part of it enters in the bore of the instrument, than in reed instruments where all exhaled air is pushed through the bore.

One expects that all tidal air will be exhaled prior to pulmonary air (though some mixing will take place), thus (in keeping with Mr. Pay's analysis) one expects fairly short-term sharpening of pitch right after a breath, followed by stable pitch once pulmonary air is predominant, rather than a long-term gradual effect.

Finally, it is perhaps worthwhile to point out the pitfalls of linear interpolation in nonlinear relationships. As a first approximation, one could blindly linearly interpolate between experimental values of sound velocity in air, O_2, and CO_2. This would generate an incorrect muliplier of $(1-0.17p)$ instead of $(1-0.26p)$ for v. Likewise, blind linear interpolation between experimental values of γ would yield a multiplier there of $(1-0.07p)$ instead of $(1-0.10p)$. In the above analysis, linear interpolation has only been used for D, C_P and C_V, where it is appropriate by the ideal gas law and Dalton's Law of Partial Pressures. The resultant multipliers, as linear functions of p, have been propagated via differential approximation, which is valid since $|p| << 1$.

REFERENCES

[1] Antony Pay, “The mechanics of playing the clarinet”, in *The Cambridge Companion to the Clarinet*, Colin Lawson, ed., CUP, 1995.
[2] The Klarinet mailing list [email], accessible through `http://www.sneezy.org/clarinet/Klarinet/`

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CHICAGO, CHICAGO IL 60637
E-mail address: `pergler@math.uchicago.edu`